

2023



AP[®] Physics C: Mechanics

Scoring Guidelines Set 2

Question 1: Free-Response Question**15 points**

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|----------------|--|----------------|
| (a)(i) | For stating that the area bounded by the curve is the displacement | 1 point |
| (a)(ii) | For indicating momentum is conserved | 1 point |

Example Response

$$\Sigma p_0 = \Sigma p_f$$

$$m_A v_{A0} + m_B v_{B0} = m_A v_{Af} + m_B v_{Bf}$$

$$m_A v_{A0} - m_A v_{Af} = m_B v_{Bf}$$

For the correct answer for speed with units (2 m/s)	1 point
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Example Solution

$$m_A v_{A0} + m_B v_{B0} = m_A v_{Af} + m_B v_{Bf}$$

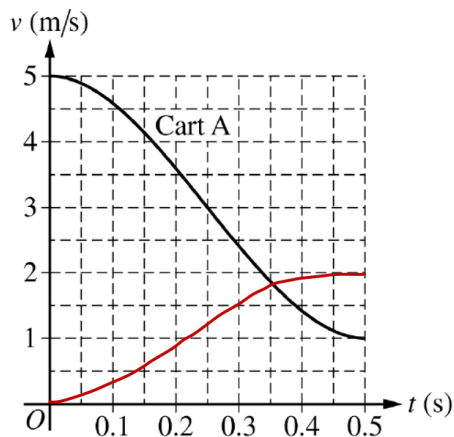
$$m_A v_{A0} - m_A v_{Af} = m_B v_{Bf}$$

$$v_{Bf} = \frac{m_A}{m_B} (v_{A0} - v_{Af})$$

$$\rightarrow v_{Bf} = \frac{1000 \text{ kg}}{2000 \text{ kg}} (5 \text{ m/s} - 1 \text{ m/s})$$

$$\therefore v_{Bf} = 2 \text{ m/s}$$

- | | | |
|-----------------|---|----------------|
| (a)(iii) | For a graph that starts at (0,0) and ends at (0.5,2) or value consistent with (a)(ii) | 1 point |
| | For a smooth, continuous curve that transitions from concave up to concave down | 1 point |

Example Solution

Total for part (a)	5 points
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(b)(i) For indicating the derivative of the velocity function is the acceleration of the cart **1 point**

Example Response

$$a(t) = \frac{dv}{dt}$$

$$\rightarrow a(t) = \frac{d(64t^3 - 48t^2 + 5)}{dt}$$

$$\therefore a(t) = 192t^2 - 96t$$

For setting the derivative of the previously derived expression for acceleration equal to zero **1 point**

Example Response

$$\frac{d}{dt}a(t) = 0$$

$$0 = 384t - 96$$

For indicating the maximum acceleration is 12 m/s^2 or maximum acceleration occurs at time **1 point**
 $t = 0.25 \text{ s}$

Example Response

$$0 = 384t - 96$$

$$\therefore t_{\text{max}} = 0.25 \text{ s}$$

For substituting the time at which the acceleration is a maximum or the maximum acceleration into an expression of Newton's second law to calculate the value of the maximum force **1 point**

Example Response

$$F(t) = ma(t)$$

$$F_{\text{max}} = ma(0.25 \text{ s})$$

Example Solution

$$a(t) = \frac{dv}{dt}$$

$$\rightarrow a(t) = \frac{d(64t^3 - 48t^2 + 5)}{dt}$$

$$\therefore a(t) = 192t^2 - 96t$$

$$\frac{d}{dt}a(t) = 0$$

$$\frac{d(192t^2 - 96t)}{dt} = 0$$

$$\rightarrow 0 = 384t - 96$$

$$\therefore t_{\max} = 0.25 \text{ s}$$

$$F(t) = ma(t)$$

$$F_{\max} = ma(0.25 \text{ s})$$

$$F_{\max} = (1000 \text{ kg})(192(0.25 \text{ s})^2 - 96(0.25 \text{ s}))$$

$$\therefore |F_{\max}| = 12,000 \text{ N}$$

Alternate Solution

For indicating the derivative of the momentum function is the force exerted on the cart **1 point**

Alternate Example Response

$$F(t) = \frac{dp}{dt}$$

$$F(t) = \frac{d}{dt}(64000t^3 - 48000t^2 + 5000)$$

For setting the derivative of the previously derived expression for force equal to zero **1 point**

Alternate Example Response

$$\frac{d}{dt}F(t) = 0$$

$$\frac{d}{dt}(192000t^2 - 96000t) = 0$$

For indicating the maximum force occurs at time $t = 0.25$ s **1 point**

Alternate Example Response

$$0 = 384000t - 96000$$

$$\therefore t_{\max} = 0.25 \text{ s}$$

For substituting the time at which the force is at a maximum into an expression for momentum to calculate the value of the maximum force **1 point**

Alternate Example Response

$$F(t) = \frac{d}{dt}p(t)$$

$$F_{\max} = \frac{d}{dt}p(0.25 \text{ s})$$

Alternate Example Solution

$$p(t) = mv(t)$$

$$\rightarrow p(t) = (1000 \text{ kg})(64t^3 - 48t^2 + 5)$$

$$\therefore p(t) = 64000t^3 - 48000t^2 + 5000$$

$$F(t) = \frac{dp}{dt}$$

$$\rightarrow F(t) = \frac{d}{dt}(64000t^3 - 48000t^2 + 5000)$$

$$\therefore F(t) = 192000t^2 - 96000t$$

$$\frac{d}{dt}F(t) = 0$$

$$\frac{d}{dt}(192000t^2 - 96000t) = 0$$

$$\rightarrow 0 = 384000t - 96000$$

$$\therefore t_{\text{max}} = 0.25 \text{ s}$$

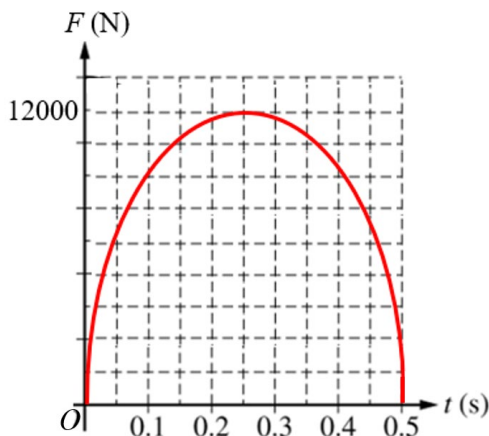
$$F(t) = \frac{d}{dt}p(t)$$

$$F_{\text{max}} = \frac{d}{dt}p(0.25 \text{ s})$$

$$F_{\text{max}} = 192000(0.25 \text{ s})^2 - 96000(0.25 \text{ s})$$

$$\therefore |F_{\text{max}}| = 12,000 \text{ N}$$

(b)(ii)	For a curve that increases and then decreases in value that is only concave down	1 point
	For a labeled maximum value consistent with the calculated value in part (b)(i)	1 point
For the graph having values of 0 N at $t = 0$ s and $t = 0.5$ s		1 point

Example Solution

Total for part (b)		7 points
(c)	For selecting $F_1 < F_2$ with an attempt at a relevant justification	1 point
	For indicating that the impulse or change in momentum of each cart in both collisions is the same	1 point
	For indicating that decreasing the time of collision means the average force must be greater	1 point

Example Solution

Since the initial and final velocities are the same for both collisions, Δp is the same for both collisions; as a result, the impulse is the same for both collisions. So, if Δt is smaller, F_{avg} is larger.

Total for part (c) **3 points**

Total for question 1 **15 points**

Question 2: Free-Response Question**15 points**

- (a) For indicating the equivalent spring constant k_{eq} is the sum of the spring constants of all the springs arranged in parallel **1 point**

Example Response

$$k_{\text{eq}} = Nk$$

For a correct expression for period consistent with k_{eq} above **1 point**

Example Response

$$T = 2\pi\sqrt{\frac{m}{Nk}}$$

Example Solution

$$k_{\text{eq}} = \sum_{i=1}^N k_i$$

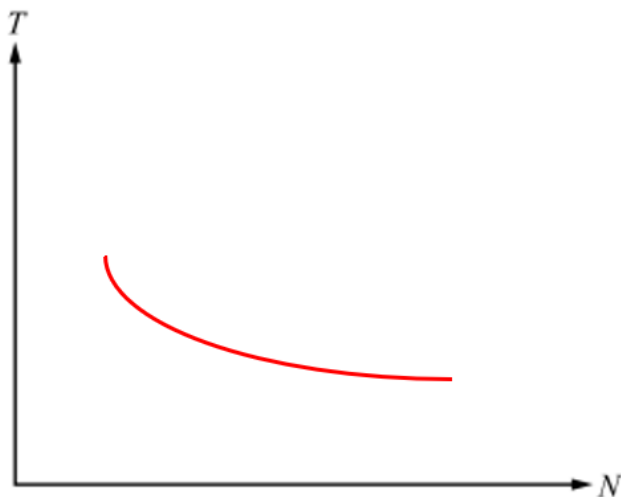
$$k_{\text{eq}} = Nk$$

$$T = 2\pi\sqrt{\frac{m}{Nk}}$$

Total for part (a) 2 points

- (b) For a graph that increases or decreases with N consistent with the expression derived in part (a) **1 point**

For a concavity that is consistent with the expression derived in part (a) **1 point**

Example Solution

Scoring Note: The student is not required to label $N = 1$ on the horizontal axis, so the curve can start at the vertical axis, implying that $N = 1$ is at the origin.

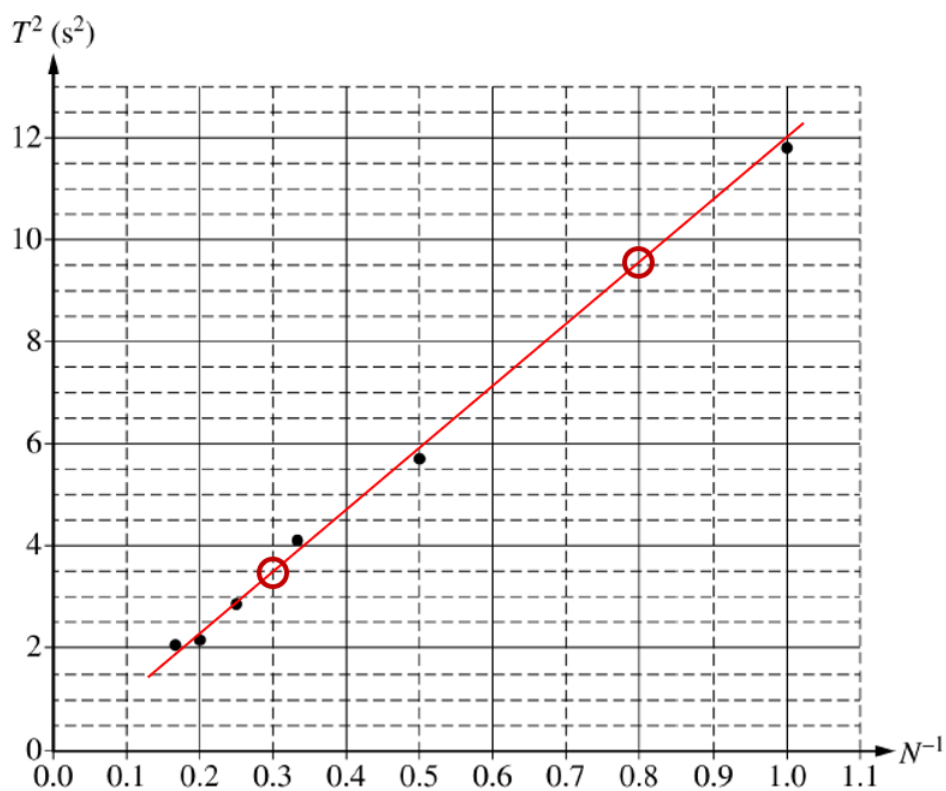
Scoring Note: Discrete points following the appropriate curve will earn this point.

Total for part (b) 2 points

(c)(i) For drawing an appropriate line of best fit that approximates the data

1 point

Example Solution



(c)(ii) For using two points from the best-fit line to calculate slope 1 point

Scoring Note: Using data points that fall on the best-fit line is acceptable.

Example Response

$$\text{slope} = \frac{\Delta(T^2)}{\Delta(N^{-1})}$$

$$\text{slope} = \frac{(9.5 \text{ s}^2 - 3.5 \text{ s}^2)}{(0.8 - 0.3)}$$

$$\therefore \text{slope} = 12 \text{ s}^2$$

For relating the slope of the line of the T^2 vs. N^{-1} graph consistent with the expression derived in part (a) 1 point

Example Response

$$T^2 = 4\pi^2 \frac{m}{Nk}$$

$$\text{slope} = T^2 N$$

$$\text{slope} = 4\pi^2 \frac{m}{k}$$

For a calculated answer that has units of N/m 1 point

Example Response

$$k = 4.93 \text{ N/m}$$

Example Solution

$$T^2 = 4\pi^2 \frac{m}{Nk}$$

$$\text{slope} = 4\pi^2 \frac{m}{k}$$

$$k = \frac{4\pi^2 m}{\text{slope}}$$

$$k = \frac{4\pi^2 (1.5 \text{ kg})}{(9.5 \text{ s}^2 - 3.5 \text{ s}^2) / (0.8 - 0.3)}$$

$$\rightarrow k = \frac{4\pi^2 (1.5 \text{ kg})}{12 \text{ s}^2}$$

$$k = 4.93 \text{ N/m}$$

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|-----------------|---|----------------|
| (c)(iii) | For indicating a source of error that could result in the observed difference with an attempt at a relevant justification | 1 point |
|-----------------|---|----------------|

Examples include:

- Experimental uncertainties in the mass of the system
- Motion detector miscalibration
- Timing error due to reaction time

For a correct justification that links the source of experimental error to the smaller experimental value of k	1 point
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Examples include:

- Experimental uncertainties in the mass of the system (e.g., mass of block is too small, not accounting for the mass of the spring)
- Calibration of the motion sensor produces a graph from which too large a period is measured
- A larger period due to measurement error will result in a smaller value of k

	Total for part (c) 6 points
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|---------------|--|----------------|
| (d)(i) | For indicating that the slope would not change with an attempt at a relevant justification | 1 point |
|---------------|--|----------------|

For a justification that the period depends on the mass and spring constant which remain unchanged	1 point
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OR

For a justification that indicates that the period is independent of gravitational force

Example Solution

The period of oscillation of the spring-block system depends on the mass and the effective spring constant, $T = 2\pi\sqrt{\frac{m}{Nk}}$. The slope is equal to the square of the period over the

inverse number of identical springs, $\text{Slope} = \frac{T^2}{N^{-1}}$. This means that the slope is proportional to the mass divided by the spring constant. Since neither the mass nor the spring constant change, the slope will remain unchanged.

OR

The period of oscillation of the spring-block system depends on the mass and the effective spring constant, $T = 2\pi\sqrt{\frac{m}{Nk}}$. The slope is equal to the square of the period over the

inverse number of identical springs, $\text{Slope} = \frac{T^2}{N^{-1}}$. Since the period is independent of the gravitational force, and the only change was arranging the block-spring system horizontally instead of vertically, the period is not affected by the change in the orientation of the system. Therefore, the slope will remain unchanged.

(d)(ii)	For a relationship between $v_{\max}$ and N that is consistent with the expression from part (a) with an attempt at relevant justification	1 point
	For using energy conservation to justify the relationship	1 point

ORFor using the relationship between $v_{\max}$ and ω to justify the relationship

For indicating that the effective spring constant changes the elastic potential energy of the spring block system U_s therefore changing $v_{\max}$ in a manner consistent with the expression from part (a)	1 point
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ORFor an inverse relationship between ω and period, indicating a change in $v_{\max}$ consistent with the expression from part (a)**Example Solution**

An increase in the number of identical springs attached in parallel to each other causes the effective spring constant to increase, $k_{\text{eff}} = \sum_{i=1}^N k_i$. Since the effective spring constant is

proportional to potential energy of the block-spring system, $U_s = \frac{1}{2}k_{\text{eff}}x^2$, an increase in the effective spring constant causes the potential energy of the block-spring system to increase for a given displacement. Therefore, based on conservation of energy,

$$\frac{1}{2}k_{\text{eff}}x^2 = \frac{1}{2}mv_{\max}^2$$

, an increase in the potential energy will cause an increase in the kinetic energy, resulting in a greater maximum velocity. Therefore, the maximum velocity will increase with an increase in the number of springs added.

OR

An increase in the number of identical springs attached in parallel to each other causes the effective spring constant to increase, $k_{\text{eff}} = \sum_{i=1}^N k_i$. Since the effective spring constant is

inversely proportional to the period of oscillation of the block-spring system, $T = 2\pi\sqrt{\frac{m}{k_{\text{eff}}}}$, an increase in the effective spring constant causes the period to decrease. Since the period is inversely proportional to the angular frequency of the block-spring system, $\omega = \frac{2\pi}{T}$, the angular frequency increases. Since the maximum velocity is proportional to the angular frequency, $v_{\max} = x_0\omega$, this results in an increase in the maximum velocity. Therefore, the maximum velocity will increase with an increase in the number of springs added.

Total for part (d) 5 points**Total for question 2 15 points**

Question 3: Free-Response Question**15 points**

- (a) For stating the parallel axis theorem **1 point**

Example Response

$$I_{\text{blade}} = I_{\text{CM}} + Md^2$$

For using correct substitutions of the rotational inertia of one blade about its center of mass and substituting the distance from the center of mass **1 point**

Example Response

$$I_{\text{blade}} = \frac{1}{18} ML^2 + M \left(\frac{L}{3} \right)^2$$

For multiplying the rotational inertia of one blade by 3 **1 point**

Example Response

$$I_{\text{rotor}} = 3I_{\text{blade}} = \frac{1}{2} ML^2$$

Example Solution

$$I_{\text{blade}} = I_{\text{CM}} + Md^2$$

$$I_{\text{blade}} = \frac{1}{18} ML^2 + M \left(\frac{L}{3} \right)^2$$

$$I_{\text{blade}} = \frac{1}{6} ML^2$$

$$I_{\text{rotor}} = 3I_{\text{blade}} = \frac{1}{2} ML^2$$

Total for part (a) 3 points

- (b) For calculating the correct answer with correct units (2.4 s) **1 point**

Example Solution

$$v = r\omega$$

$$v = L\omega_0$$

$$\frac{d}{t} = L\omega_0$$

$$t = \frac{d}{L\omega_0} = \frac{2\pi L}{L\omega_0} = \frac{2\pi}{\omega_0}$$

$$t = \frac{2\pi}{(2.6 \text{ rad/s})}$$

$$\therefore t = 2.4 \text{ s}$$

Total for part (b) 1 point

(c)(i) For indicating that the total initial rotational kinetic energy is dissipated **1 point**

Example Response

$$\Delta K_{\text{rot}} = E_{\text{dis}}$$

$$0 - \frac{1}{2} I \omega_0^2 = E_{\text{dis}}$$

For substituting correct values for the rotational inertia and initial angular speed of the system **1 point**

Example Response

$$E_{\text{dis}} = -\frac{1}{2} (6.7 \times 10^6 \text{ kg} \cdot \text{m}^2) (2.6 \text{ rad/s})^2$$

For an answer consistent with substitutions above and with correct units **1 point**

Example Response

$$E_{\text{dis}} = -2.3 \times 10^7 \text{ J}$$

Example Solution

$$\Delta K_{\text{rot}} = E_{\text{dis}}$$

$$0 - \frac{1}{2} I \omega_0^2 = E_{\text{dis}}$$

$$E_{\text{dis}} = -\frac{1}{2} (6.7 \times 10^6 \text{ kg} \cdot \text{m}^2) (2.6 \text{ rad/s})^2$$

$$E_{\text{dis}} = -2.3 \times 10^7 \text{ J}$$

Scoring Note: A response may earn full credit for positive or negative values of dissipated energy.

(c)(ii)	For using Newton’s second law in rotational form	1 point
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	For attempting to differentiate the equation for ω	1 point
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Example Response

$$\tau = I_{\text{sys}} \frac{d}{dt} (\omega_0 e^{-\beta_0 t})$$

	For a correct expression for the torque on the system	1 point
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Example Response

$$\tau = -\beta_0 I_{\text{sys}} \omega_0 e^{-\beta_0 t}$$

Example Solution

$$\tau = I\alpha$$

$$\tau = I_{\text{sys}} \frac{d\omega}{dt}$$

$$\tau = I_{\text{sys}} \frac{d}{dt} (\omega_0 e^{-\beta_0 t})$$

$$\therefore \tau = -\beta_0 I_{\text{sys}} \omega_0 e^{-\beta_0 t}$$

(c)(iii)	For attempting to integrate the expression for angular speed	1 point
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	Example Response	
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$$\Delta\theta = \int \omega(t) dt$$

	For using the correct limits of integration	1 point
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	Example Response	
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$$\Delta\theta = \int_0^t \omega_0 e^{-\beta_0 t} dt$$

	For a correct expression for angular displacement	1 point
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	Example Response	
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$$\Delta\theta = \frac{\omega_0}{\beta_0} (1 - e^{-\beta_0 t})$$

	Example Solution	
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$$\Delta\theta = \int \omega(t) dt$$

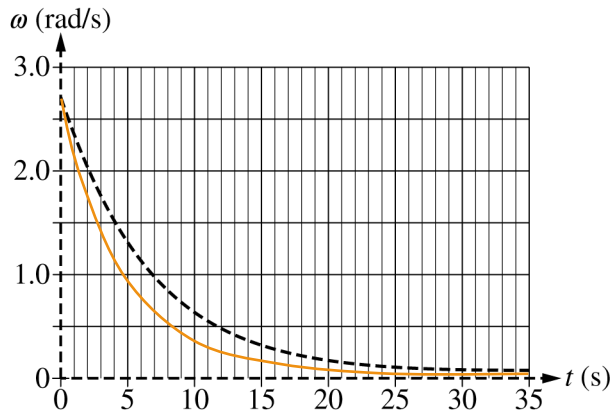
$$\Delta\theta = \int_0^t \omega_0 e^{-\beta_0 t} dt$$

$$\Delta\theta = \left(-\frac{\omega_0}{\beta_0} e^{-\beta_0 t} \right) \bigg|_0^t = -\frac{\omega_0}{\beta_0} e^{-\beta_0 t} + \frac{\omega_0}{\beta_0} (1)$$

$$\therefore \Delta\theta = \frac{\omega_0}{\beta_0} (1 - e^{-\beta_0 t})$$

	Total for part (c)	9 points
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(d)	For drawing a continuous curve showing an exponential decay	1 point
	For starting at $\omega = 2.6 \text{ rad/s}$ and drawing a curve below the original curve	1 point

Example Solution

Total for part (d) 2 points

Total for question 3 15 points